Identifying structural shocks to volatility through a proxy-MGARCH model

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1. Motivation

- Multivariate volatility models (MVMs) are good at describing stylized facts of asset returns and widely used in modeling and forecasting second order moment dynamics.
- As a drawback, they do not provide an intelligible interpretation of the shock system driving asset returns – they are "reduced form models" as opposed to structural models as e.g. SVAR models.
- We identify the reduced form MGARCH model through a structural approach using proxy variables and Givens rotations.

2. Data

- We analyse the return system of S&P500, 10 year treasury note yield and Finex USD index from 1998 to 2014.
- Proxy variables are news indicators from Thompson Reuters: U.S. market and U.S. bond market sentiment.

3. Structural Identification Problem in MVMs

- We consider a system of n speculative log returns $r_t = \mu_t + \varepsilon_t$, $t \in I := \{1, \dots, T\}$) where $\mu_t = \mathrm{E}[r_t|\mathcal{F}_{t-1}]$ with $\mathcal{F}_t$ the σ-algebra generated by the returns up to and including time t.
- The reduced form innovations follow an MGARCH model: $\varepsilon_t|\mathcal{F}_{t-1} \sim (0, H_t)$. They do not bear an economic interpretation. Let ε_t be generated by

$$\varepsilon_t|\mathcal{F}_{t-1} \sim Q_t \xi_t \ , \tag{1}$$

where $(\xi_t)_{t\in I}$ is an n-dimensional vector of structural shocks with $\mathrm{E}[\xi_t] = 0$ and $\mathrm{E}[\xi_t \xi_t^\top] = I_n$.

- Q_t denotes the unknown structural matrix decomposition of H_t which satisfies $Q_t Q_t^\top = H_t$.
- Given the principal matrix square root as initial decomposition $\tilde{Q}_t$, we identify the true structural matrix decomposition Q_t by identifying the unique rotation $\tilde{R}$ such that $\tilde{Q}_t \tilde{R} = Q_t$. The structural model parameters are given by the rotation matrix.
- Our identification problem differs from the SVAR case with modeled heteroscedasticity as we account for complex dynamics in the conditional covariance process of ε_t. E.g. $\tilde{Q}_t$ thus varies with H_t over time.

4. Identification by Proxy and Orthogonalization by Givens Rotations

- Identification similar to the proxy-SVAR approach of Mertens and Ravn (2013); Stock and Watson (2012). Assume there exists a centered $(n-1)$-dimensional proxy variable process $Z = (Z_t)_{t\in I}$ such that, for all $i = 1, \dots, n-1$,

$$\mathrm{E}[\xi_{it} Z_{it}] = \phi_i \in \mathbb{R} \setminus \{0\} \qquad \text{(relevance)} \tag{2}$$

$$\mathrm{E}[\xi_{jt} Z_{it}] = 0 \ (j \neq i) \qquad \text{(exogeneity)} \tag{3}$$

- Then one can estimate the columns of the rotation matrix by

$$\tilde{R}_{\bullet i} = \pm\, \mathrm{E}[u_t Z_{it}] \left(\mathrm{E}[Z_{it} u_t^\top]\, \mathrm{E}[u_t Z_{it}] \right)^{-1/2} \tag{4}$$

where u_t denote the MGARCH residuals standardized with the principal matrix square root of H_t.

- To identify the full rotation matrix we exploit that n-dimensional rotations can be expressed as sequences of Givens rotations which guarantees orthogonality of the structural shocks.
- This is of particular importance, as it is a priori not clear which proxy variables deliver orthogonal shocks.

5. Rotation & Structural Model Results

Every $\tilde{R} \in \mathbb{R}^{n\times n}$ can be expressed as a composition of $\frac{n(n-1)}{2}$ "two-dimensional" rotations $R^{ij}(\theta_{ij})$. Set

$$\tilde{R} = \prod_{i=1}^{n-1} \prod_{j=i+1}^{n} R^{ij}(\theta_{ij}) \quad (\theta_{ij} = \text{rotation angle})$$

$$= R^{12}(\theta_{12}) \cdots R^{1n}(\theta_{1n}) R^{23}(\theta_{23}) \cdots R^{n-1,n}(\theta_{n-1,n})$$

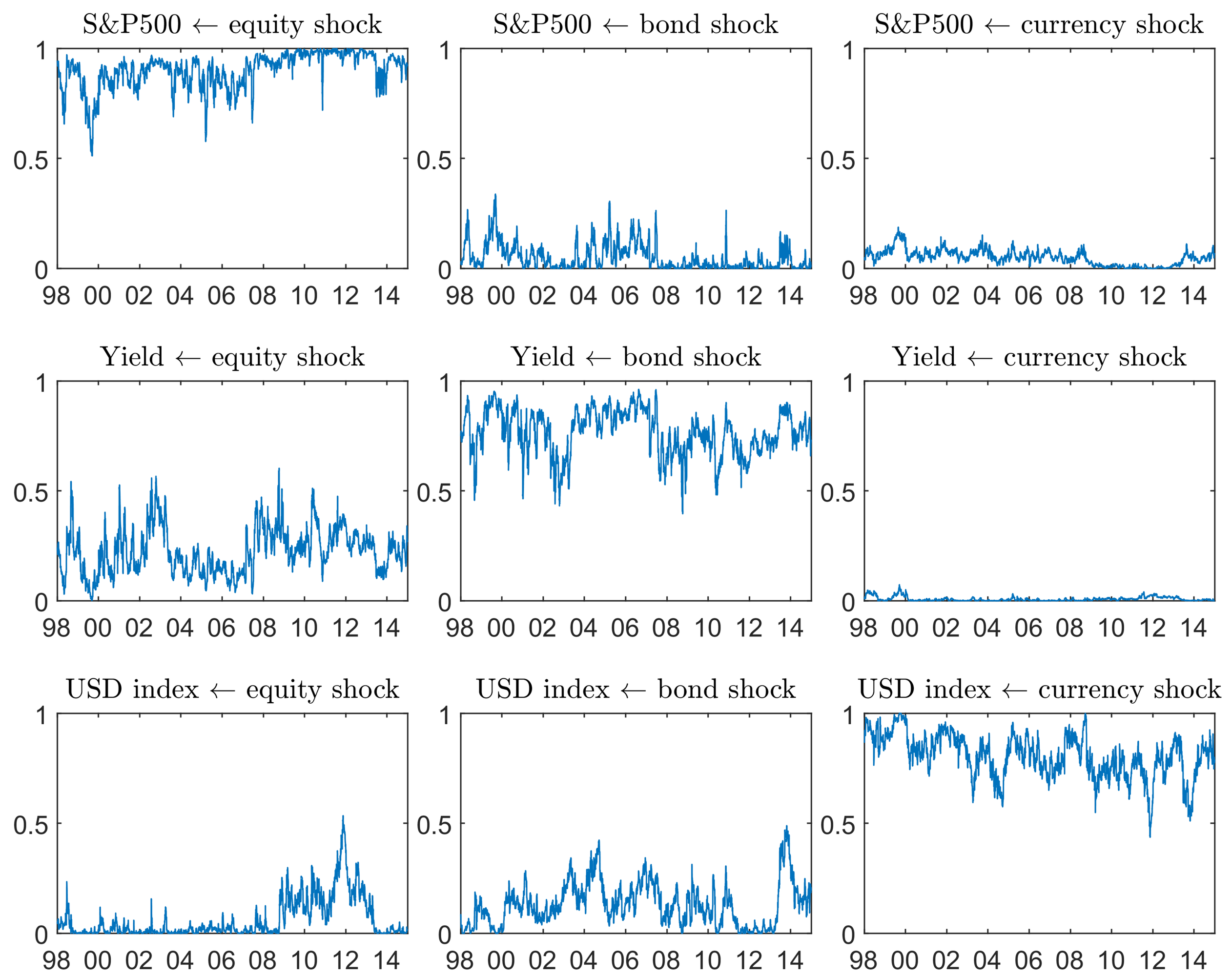
Knowledge of the first column of $\tilde{R}$ allows us to infer the first $(n-1)$ rotation angles and we are left with the identification of a $(n-1)$-dimensional rotation using the remaining $(n-2)$ proxy variables. We can identify the full rotation recursively.

Estimation Results

		proxy-MGARCH model		
$(\hat{\theta}_{12}, \hat{\theta}_{13}, \hat{\theta}_{23})^\top$		0.3811	−0.1885	−2.9164
$\hat{\tilde{R}}$		0.9118	0.3238	−0.2526
		0.3654	−0.9204	0.1393
		−0.1874	−0.2193	−0.9575
		ξ_1	ξ_2	ξ_3
correlations	Z_1	0.3347	0.0000	−0.0000
	Z_2	0.0052	0.1936	0.0000
p-values	Z_1	0	1.0000	1.0000
	Z_2	0.7286	0	1.0000
Wald test	distribution	statistic	critical value	p-value
symmetric spillovers	$\chi^2_{(4)}$	107.85	9.4877	0.0000

6. Volatility Spillover Analysis

Volatility Reception



- Volatility reception = proportional impact of the structural shocks (equity, bond & currency shock) on each asset (row-wise for S&P500, 10 yr note yield and USD index).
- Equity shock accounts for the largest share in S&P500 volatility, but the bond market shock shows notable contributions especially during calm market periods (see (1)).
- Return on the yield shows pronounced volatility reception from equity shock ((2)).
- Volatility transmission = proportional impact of one structural shock on all assets (columnwise for each structural shock).
- Long-lasting trend in volatility transmission between equity and fixed income markets (see (3) and (4))

7. Conclusion

- Provide a fully identified asset return system.
- Inferred structural shocks can be narratively corroborated in detail, they are economically meaningful, interpretable shocks.
- The volatility spillover mechanism is asymmetric. Labelled structural shocks allow us to monitor spillover patterns and directions.
- Structural volatility models open new doors, e.g. to meaningful impulse response analyses.

References

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