Nonlinear Dynamic Factor Models

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Nonlinear Dynamic Factor Model

- We introduce a **nonlinear dynamic factor model.**
  - **Nonlinear** relationship between the factors today and their past values.
  - **Nonlinear** relationship between the observables and the factors.

- Our nonlinear dynamic factor model is inspired by the pruned second-order state-space model of Kim et. al. (2008) and Andreasen, Fernandez-Villaverde, Rubio-Ramirez (2017).

- The model can generate novel implications both in terms of impulse response functions (IRFs) and predictive distributions.
  - Asymmetric and state-dependent IRFs (Andreasen, Fernandez-Villaverde, Rubio-Ramirez, 2017)
  - Non-normal predictive distributions that feature time-varying volatility and asymmetric tail behavior.

- **Empirical application:** Estimate the common component of European Credit Default Swaps (CDS) spreads from Dec 2004 - Sept 2019.

Nonlinear Dynamic Factor Model

- We take a pruned second-order approximation to a general nonlinear relationship:

$$f_t = \mathcal{H}(f_{t-1}) + \sigma\nu_t,$$

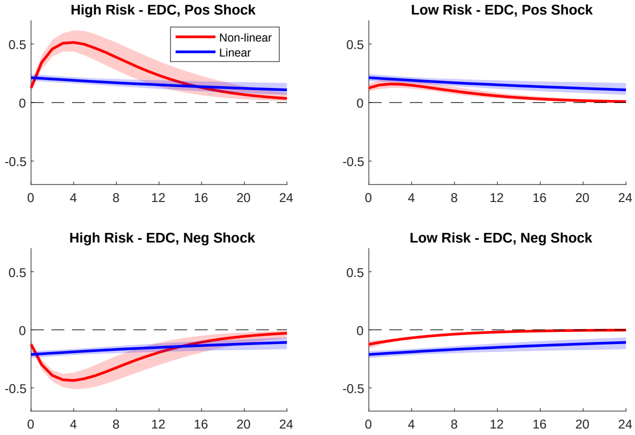
motivated by the work of Kim et. al. (2008), Andreasen et. al. (2017) and Aruoba et. al. (2017).

$$Y_t = Gf_t + e_t$$

$$\begin{aligned} f_t &= c + f_t^f + f_t^s \\ f_t^f &= h_x f_{t-1}^f + \sigma\nu_t \\ f_t^s &= h_x f_{t-1}^s + 0.5 h_{xx} \left(f_{t-1}^f \times f_{t-1}^f \right) \end{aligned}$$

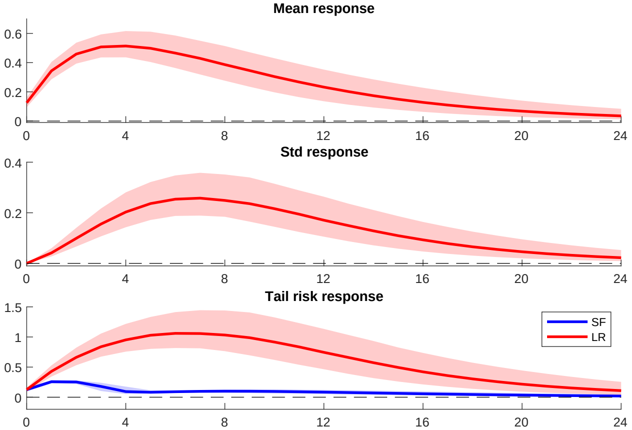
$$\begin{aligned} e_t &\sim N(0, \Omega_e) \\ \nu_t &\sim N(0, I) \end{aligned}$$

European Debt Crisis: State-Dependent Impulse Response Functions



- Nonlinear model IRFs are state-dependent and asymmetric.
- Linear model IRFs are not state-dependent and are symmetric.

Higher-order moment responses, High Risk



- Positive shocks persistently increase the mean and standard deviations of the factor distribution.
- The simultaneous increase in mean and volatility generates a persistent rise in the longrise, while the shortfall is less affected.