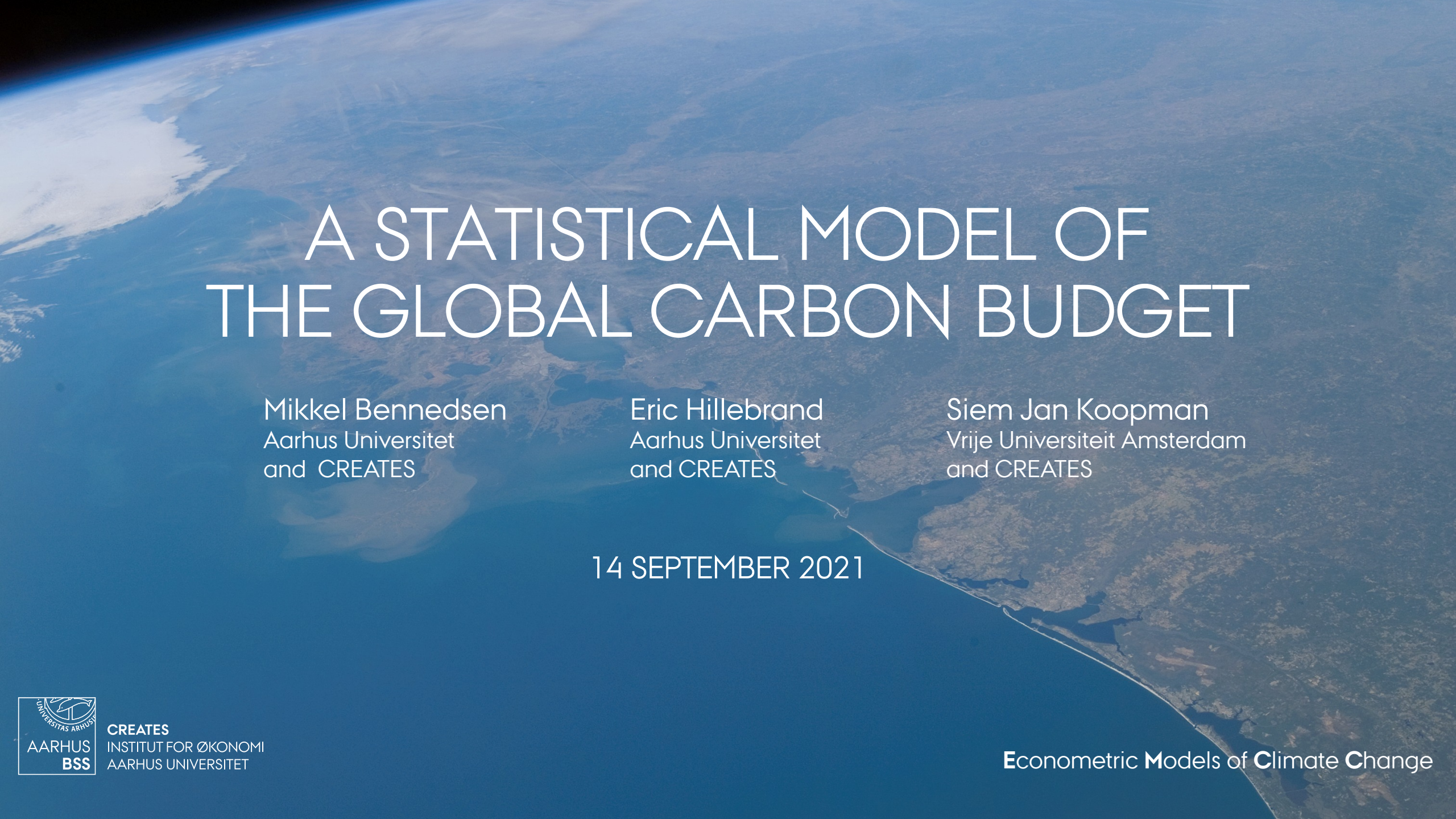

A STATISTICAL MODEL OF THE GLOBAL CARBON BUDGET

Mikkel Bennedsen
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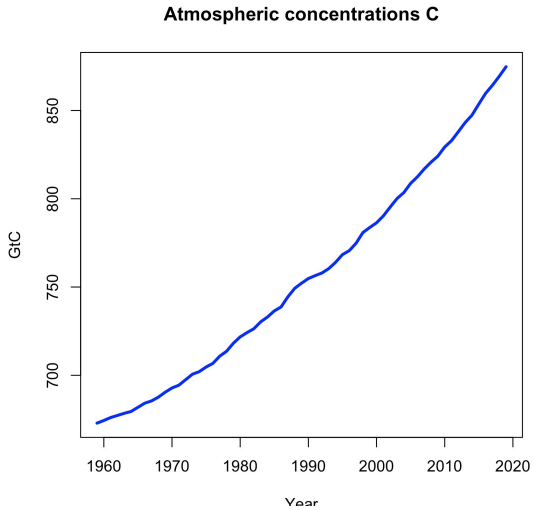
14 SEPTEMBER 2021

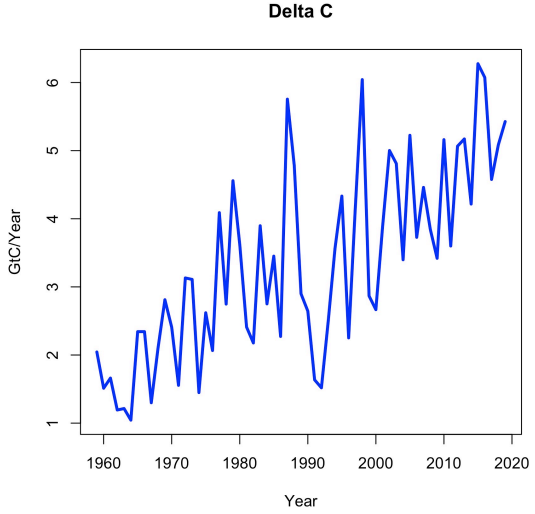
CREATES
INSTITUT FOR ØKONOMI
AARHUS UNIVERSITET

Econometric Models of Climate Change

DATA

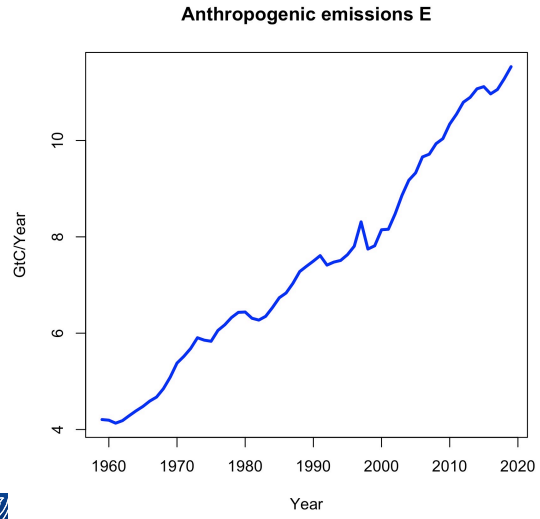
GLOBAL CARBON PROJECT

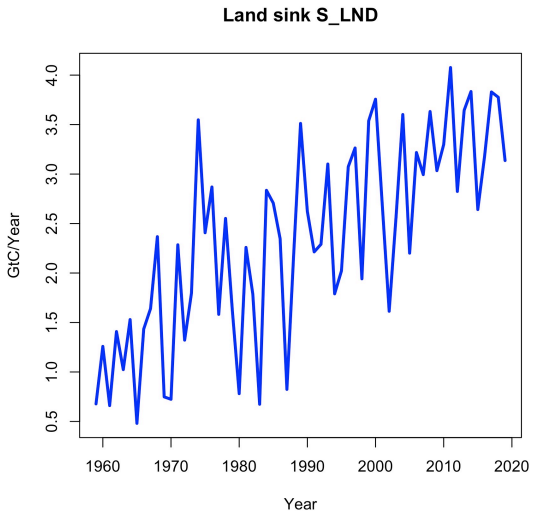
Δ 

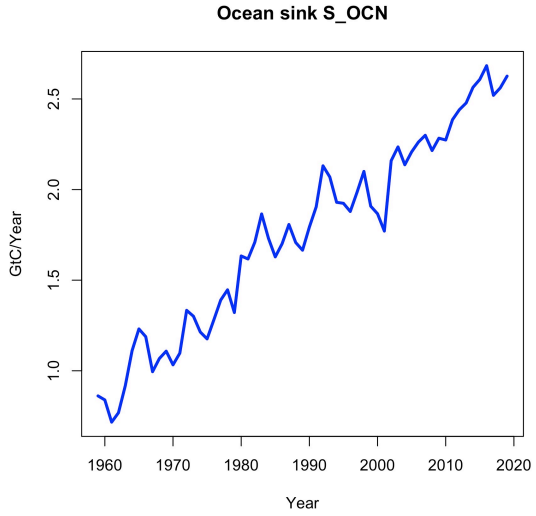
= 

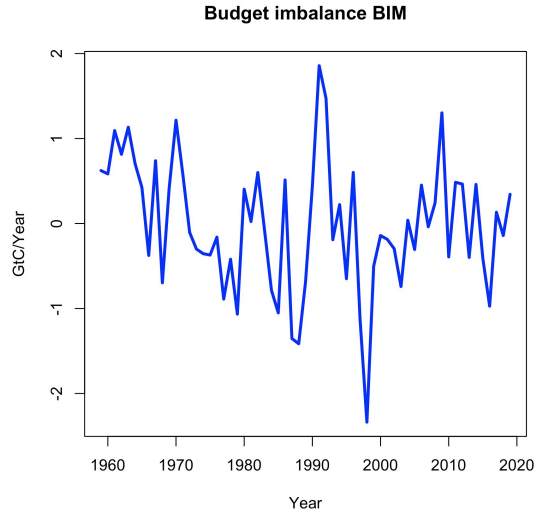
www.globalcarbonproject.org

Friedlingsstein et al. (2020),
The gobal carbon budget 2020,
Earth System Science Data 12,
3269-3340

= 

- 

- 

+ 

THE SYSTEM MODEL

State equation **Model 1**

$$S_LND^*_{t+1} = c_1 + \frac{\beta_1}{C_0} C^*_{t+1}$$

$$S_OCN^*_{t+1} = c_2 + \frac{\beta_2}{C_0} C^*_{t+1}$$

$$E^*_{t+1} = E^*_t + d + X^E_t$$

State equation **Model 2**

$$S_LND^*_{t+1} = c_1 + \frac{\beta_1}{C_0} C^*_{t+1} + \beta_3 SOI_{t+1}$$

$$S_OCN^*_{t+1} = c_2 + \frac{\beta_2}{C_0} C^*_{t+1} + \beta_4 SOI_{t+1}$$

$$E^*_{t+1} = E^*_t + \beta_5 \Delta\, GDP^{World}_{t+1} + \beta_8 I1991 + X^E_t$$

Measurement equation

$$C_t = C^*_t + X_{1,t}$$

$$S_LND_t = S_LND^*_t + X_{2,t}$$

$$S_OCN_t = S_OCN^*_t + X_{3,t}$$

$$E_t = E^*_t + \beta_6 I1997$$

$$C^*_{t+1} = C^*_t + G_ATM^*_{t+1}$$

$$G_ATM^*_{t+1} = E^*_{t+1} - S_{LND}{}^*_{t+1} - S_{OCN}{}^*_{t+1} + \beta_7 I1991$$

$$X_{1,t} = \phi_1 X_{1,t-1} + \eta_{1,t}$$

$$X_{2,t} = \eta_{2,t}$$

$$X_{3,t} = \phi_3 X_{3,t-1} + \eta_{3,t}$$

$$X^E_t = \phi_E X^E_{t-1} + \eta_{4,t}$$

$$\begin{bmatrix} \eta_{1,t} \\ \eta_{2,t} \\ \eta_{3,t} \\ \eta_{4,t} \end{bmatrix} \sim N\left(0, \begin{bmatrix} \sigma_1^2 & r_{12}\sigma_1\sigma_2 & r_{13}\sigma_1\sigma_3 & 0 \\ r_{12}\sigma_1\sigma_2 & \sigma_2^2 & 0 & 0 \\ r_{13}\sigma_1\sigma_3 & 0 & \sigma_3^2 & 0 \\ 0 & 0 & 0 & \sigma_4^2 \end{bmatrix}\right)$$

$$\eta_{4,t} \sim N(0, \sigma_4^2\, s_E^2\, I_{t\geq 1996})$$

THE DYNAMICS OF C

$$\Delta C_t = E_t - S_LND_t - S_OCN_t$$

$$= E_t - c_1 - c_2 - \beta_1^* C_t - \beta_2^* C_t + \varepsilon_t, \quad \varepsilon_t \sim I(0)$$

$$(1 + \beta_1^* + \beta_2^*) C_t - C_{t-1} = c + dt + x_t + \varepsilon_t$$

$$(1 - qL) C_t = qc + qdt + qx_t + q\varepsilon_t$$

$$\beta_i^* = \frac{\beta_i}{C_0} \approx 0.01$$

$$x_t = \sum_{i=1}^{t} X_i^E$$

$$q := \frac{1}{1 + \beta_1^* + \beta_2^*} \approx \frac{1}{1.02}$$

$$c = E_0 - c_1 - c_2$$

Three insights:

$$C_t = q^t \left[C_0 - \frac{qc}{1-q} + \frac{dq^2}{(1-q)^2} \right] + \left[\frac{qc}{1-q} - \frac{dq^2}{(1-q)^2} \right] + \frac{dq}{1-q} t + \sum_{j=0}^{t-1} q^{j+1} x_{t-j} + \sum_{j=0}^{t-1} q^{j+1} \varepsilon_{t-j}$$

$$= o(1) + O(1) + O(t) + I(1) + I(0) = O(t) + I(1)$$

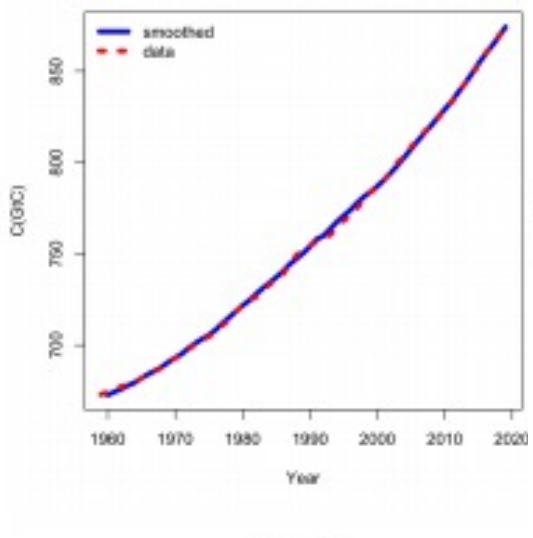
Thus,

$$\Delta C_t = I(0)$$

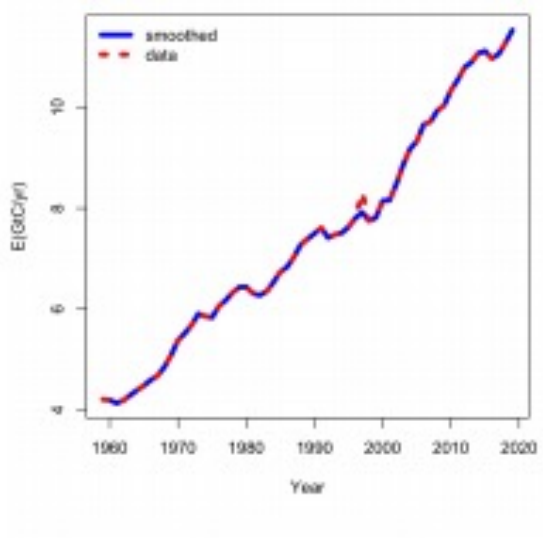
But,

$$(1 - qL)(1 - L) C_t = qd + q\Delta x_t + q\Delta \varepsilon_t = I(0)$$

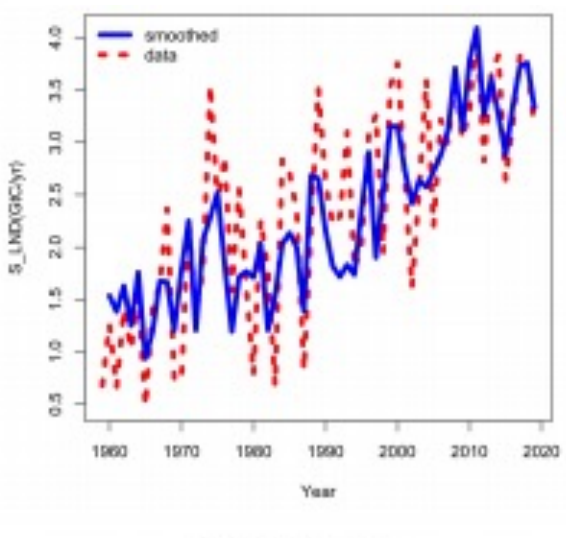
ESTIMATION



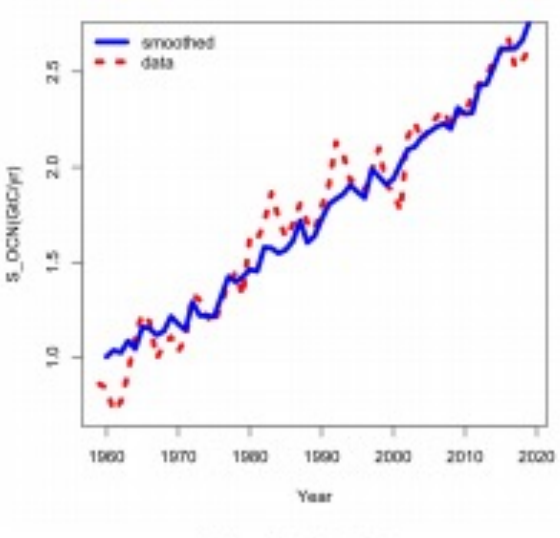
(A) C_t



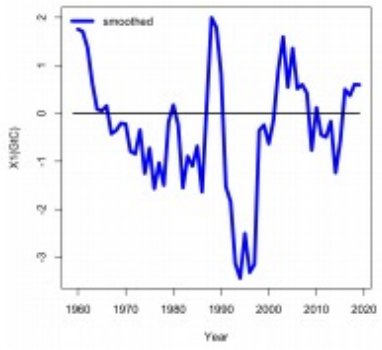
(H) E_t



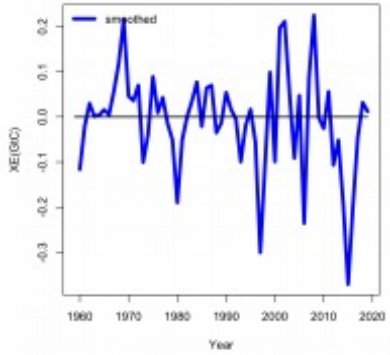
(D) S_LND_t



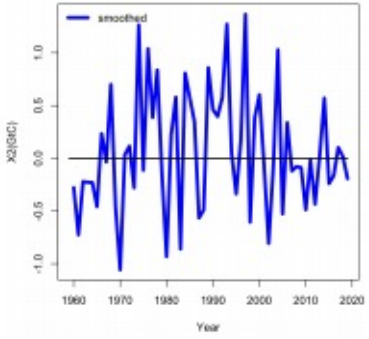
(F) S_OCN_t



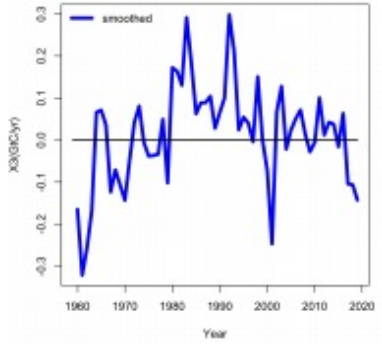
(B) $X_{1,t}$



(I) X_t^E

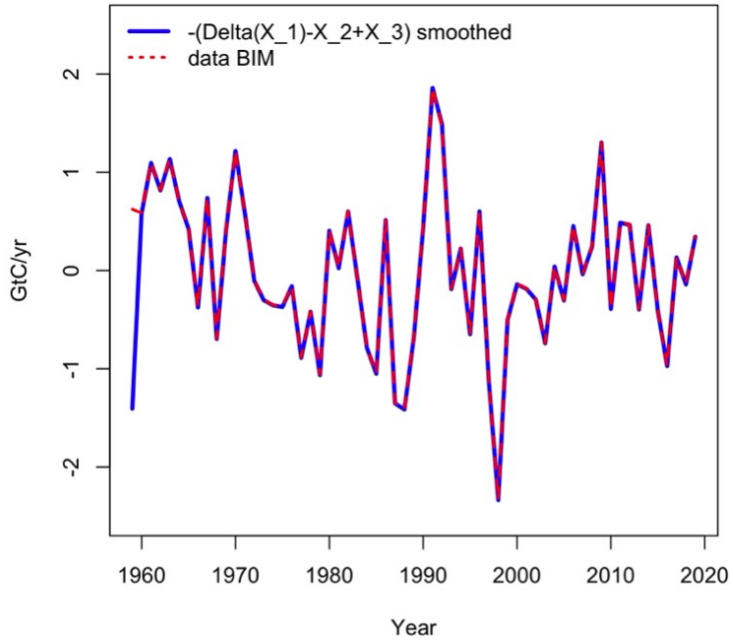


(E) $X_{2,t}$

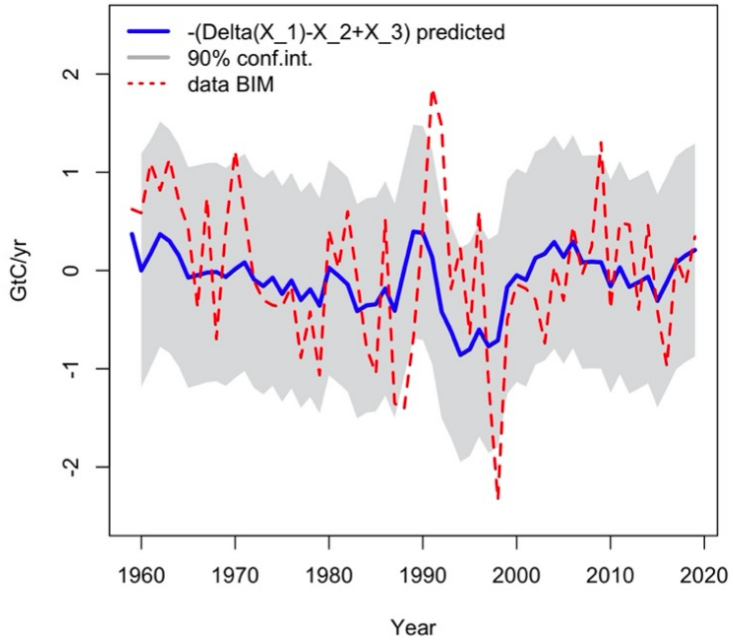


(G) $X_{3,t}$

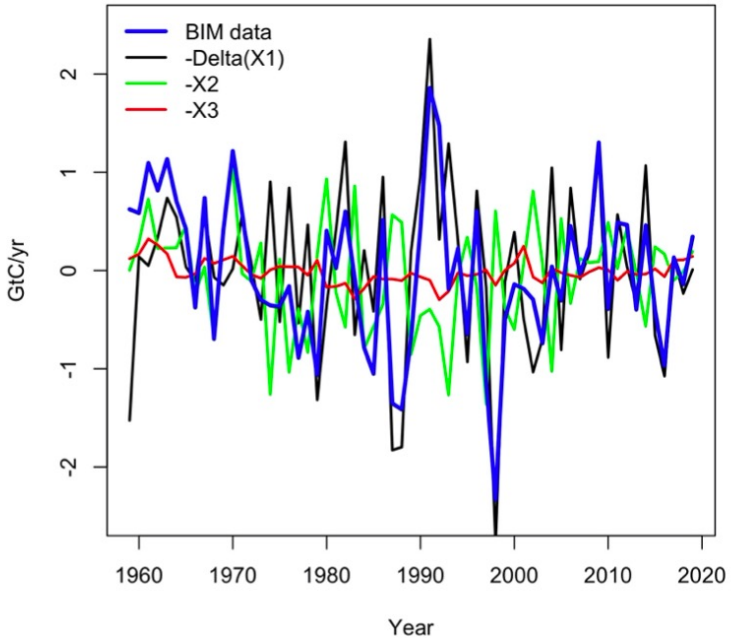
BUDGET IMBALANCE



(A) Smoothed $-(\Delta X_1 + X_2 + X_3) + \beta_6 I1997 - \beta_7 I1991$



(B) One-year ahead predictions

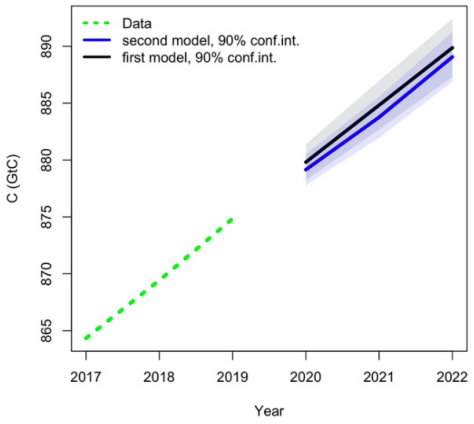


(C) Components $-\Delta X_1$, $-X_2$, $-X_3$

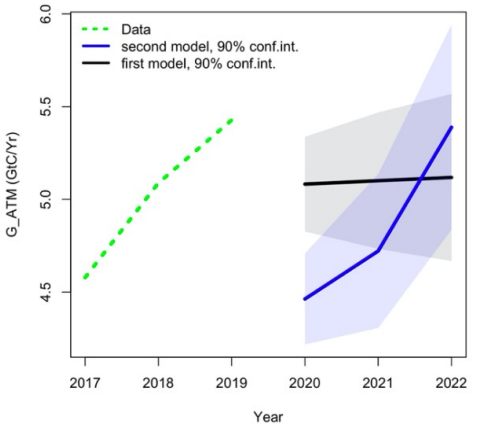
NOW-/FORECASTS

Forecasts of World GDP growth from IMF and World Bank

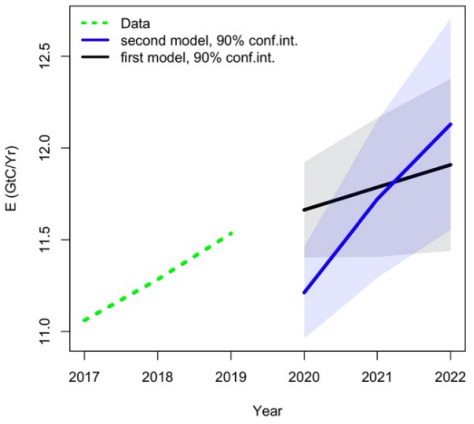
	2020	2021	2022
IMF	-3.5%	5.5%	4.2%
World Bank	-4.3%	4.0%	3.8%



(A) C - IMF

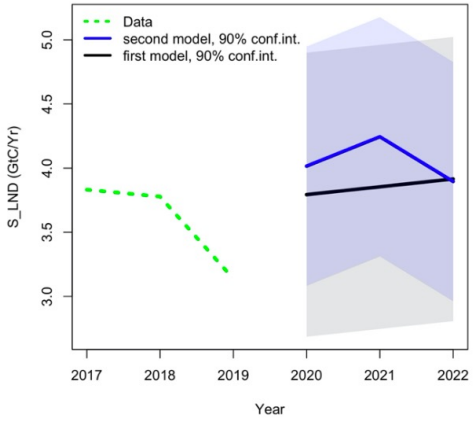


(B) G_ATM* - IMF

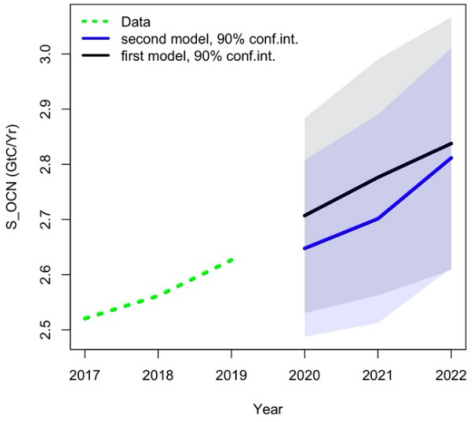


(C) E - IMF

Forecasts of SOI from forecast model of monthly SOI data 1866-2020, with trigonometric seasonal and second-order trigonometric cycle w/ period about 4 years



(D) S_LND - IMF



(E) S_OCN - IMF

PROJECTIONS TO 2050

$$E^*_{t+1} = E^*_t + \beta_{5,t}\Delta GDP_{2010,t+1} + \beta_8 I1991 + X^E_t,$$

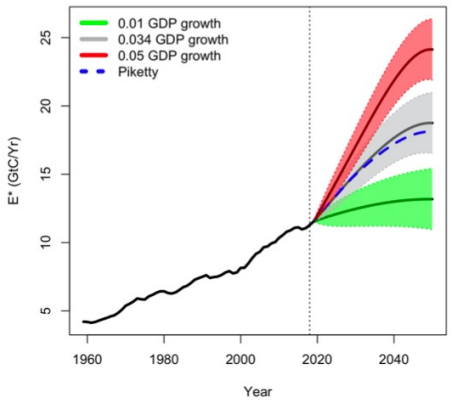
$$\beta_{5,t+1} = \beta_{5,t} + \eta_{5,t},$$

Scenarios:

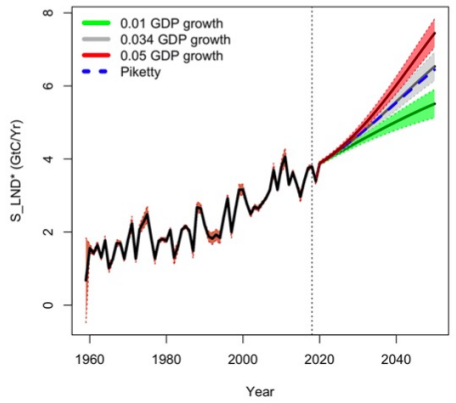
$\beta_{5,t}$ decreases linearly to 0 until 2050

No other abatement

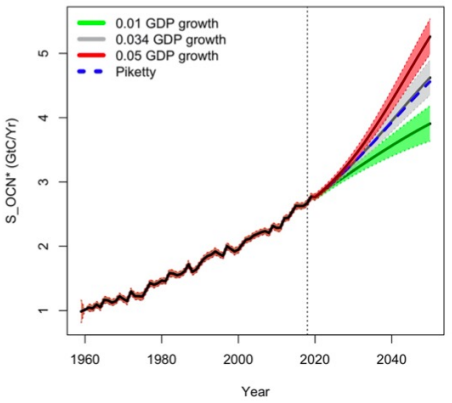
GDP
- 1% annual growth
- 3.4% annual growth
- 5% annual growth



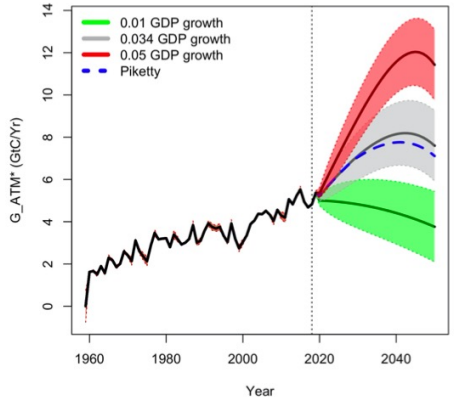
(A) E*



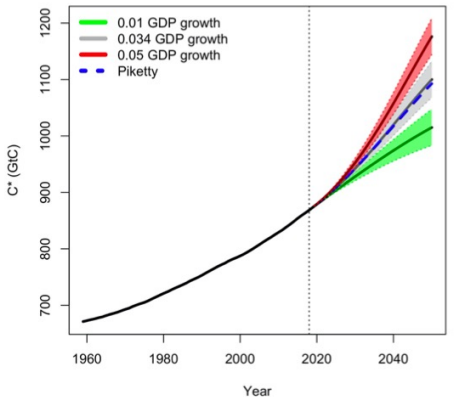
(B) S_LND*



(C) S_OCN*



(D) G_ATM*



(E) C*

IPCC SR15 (2018)
UNFCCC NDC Synthesis Report (2021)

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- Google sites: Eric Hillebrand -> Working Papers
- 45 minute video & slides

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